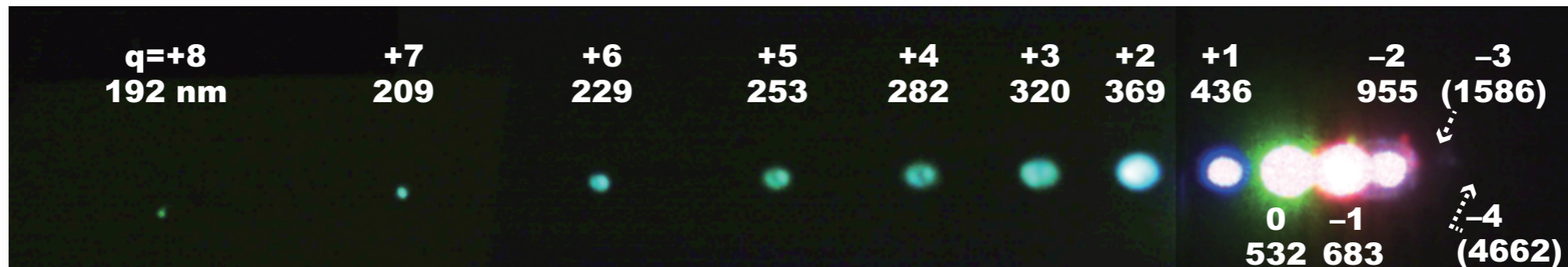


マクロコヒーレンスの 理論と実験

田中 実
大阪大学



セミナー@新潟大学, 2015/6/25

SPAN project

SPECTROSCOPY WITH ATOMIC NEUTRINO

Okayama U.

K. Kawaguchi, H. Hara, T. Masuda, Y. Miyamoto,
I. Nakano, N. Sasao, J. Tang, S. Uetake,
A. Yoshimi, K. Yoshimura, M. Yoshimura

Other institute

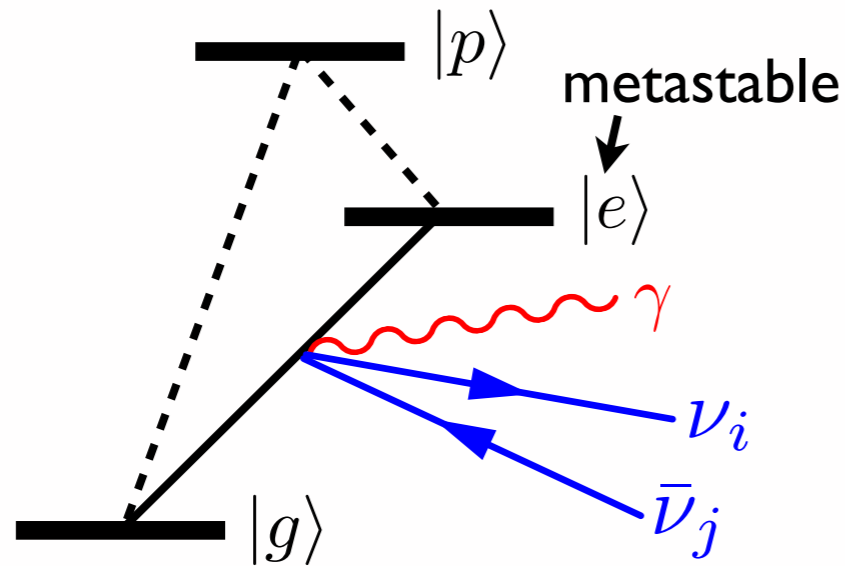
M. T. (Osaka), T. Wakabayashi (Kinki),
A. Fukumi (Kawasaki), S. Kuma (Riken),
C. Ohae (ECU), K. Nakajima (KEK), H. Nanjo (Kyoto)

INTRODUCTION

RENPN and PSR

Radiative Emission of Neutrino Pairs (RENPN)

A.Fukumi et al. PTEP (2012) 04D002, arXiv:1211.4904



$$|e\rangle \rightarrow |g\rangle + \gamma + \nu_i \bar{\nu}_j$$

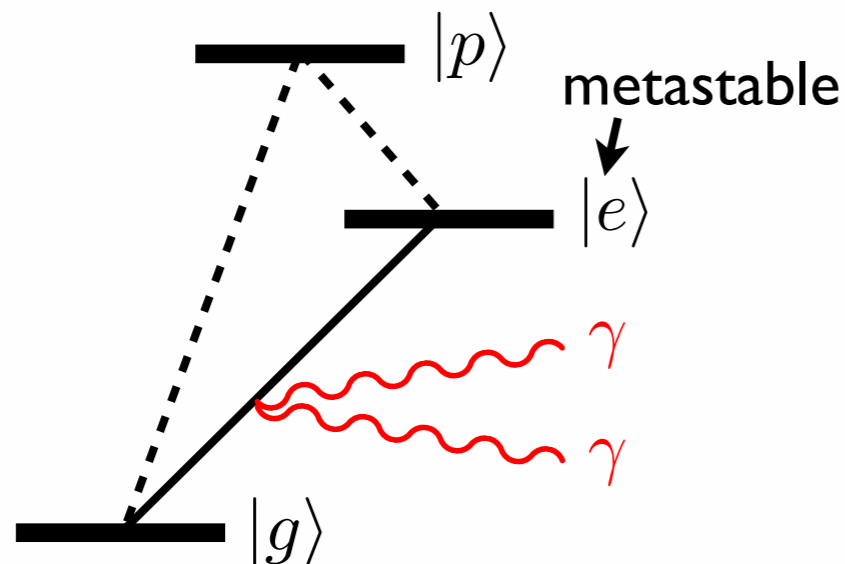
energy scale $\sim O(\text{eV})$

$$\text{Rate} \sim \alpha G_F^2 E^5 \sim 1/(10^{33} \text{ s})$$

Enhancement by coherence

Paired Super-Radiance (PSR)

M.Yoshimura, N. Sasao, MT, PRA86, 013812 (2012)



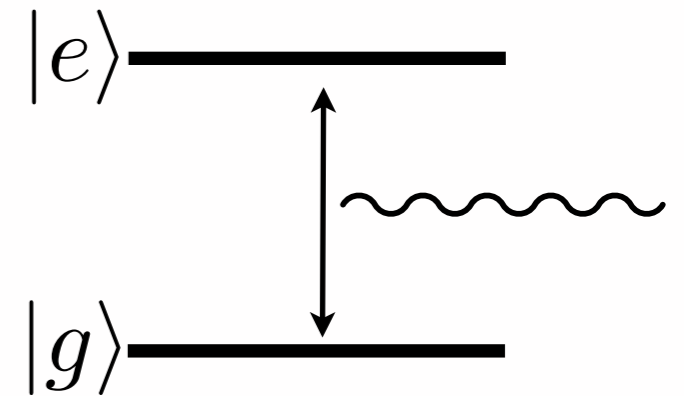
$$|e\rangle \rightarrow |g\rangle + \gamma + \gamma$$

Prototype for RENPN

proof-of-concept for
macrocoherence

Two-level system coupled to a photon

a model of emission from atoms



Hamiltonian

$$\mathcal{H} = \varepsilon_g |g\rangle\langle g| + \varepsilon_e |e\rangle\langle e| - dE |e\rangle\langle g| - d^* E^\dagger |g\rangle\langle e|$$

Density matrix (state)

$$\rho = \rho_{gg} |g\rangle\langle g| + \rho_{ee} |e\rangle\langle e| + \rho_{eg} |e\rangle\langle g| + \rho_{ge} |g\rangle\langle e|$$

Deexcitation (or emission) rate: $\Gamma = \Gamma_0 \propto |dE|^2$

Coherence (of an atomic state) ρ_{eg}

$$(|g\rangle + e^{i\varphi} |e\rangle) / \sqrt{2} \implies \rho_{eg} = e^{i\varphi} / 2, \quad |\rho_{eg}| \leq 1/2$$

Rate enhancement by coherence

R.H. Dicke,
Phys. Rev. 93, 99 (1954)

An ensemble of N atoms in a small volume L^3


$$L \ll \text{wave length} \implies e^{ikx} \sim 1$$


Fully excited state: $|e\rangle^N = |e\rangle \cdots |e\rangle$, $\rho_{eg} = 0$

deexcitation: $\left(\sum |g\rangle\langle e| \right) \prod |e\rangle$
 $= |g\rangle|e\rangle \cdots |e\rangle + |e\rangle|g\rangle \cdots |e\rangle + \cdots + |e\rangle|e\rangle \cdots |g\rangle$

 $\Gamma = N\Gamma_0$ **incoherent**

Fully coherent state: $\left[(|g\rangle + |e\rangle) / \sqrt{2} \right]^N$, $\rho_{eg} = 1/2$

 deexcitation $[|g\rangle(|g\rangle + |e\rangle) \cdots (|g\rangle + |e\rangle) + (|g\rangle + |e\rangle)|g\rangle \cdots (|g\rangle + |e\rangle) + \cdots] / \sqrt{2^N}$

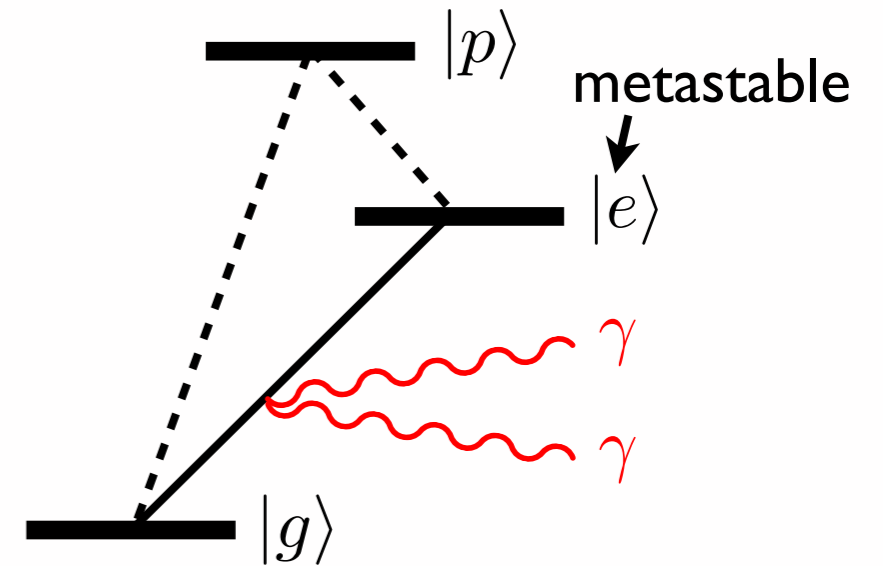
 $\Gamma = N(N+1)\Gamma_0/4 \propto N^2$ **coherent**

PSR theory

Paired Super-Radiance (PSR)

M. Yoshimura, N. Sasao, MT, PRA86, 013812 (2012)

$$|e\rangle \rightarrow |g\rangle + \gamma + \gamma$$



Prototype for RENP

proof-of-concept for the **macrocoherence**

Preparation of **initial state** for RENP

coherence generation ρ_{eg}

dynamical factor $\eta_{\omega}(t)$

Theoretical description to be tested

Maxwell-Bloch equation

PSR equation

Effective two-level interaction Hamiltonian

$$|g\rangle, |e\rangle, \cancel{|p\rangle} \quad \mathcal{H}_I = \begin{pmatrix} \alpha_{ee} & \alpha_{ge} e^{i\epsilon_{eg}t} \\ * & \alpha_{gg} \end{pmatrix} E^2$$

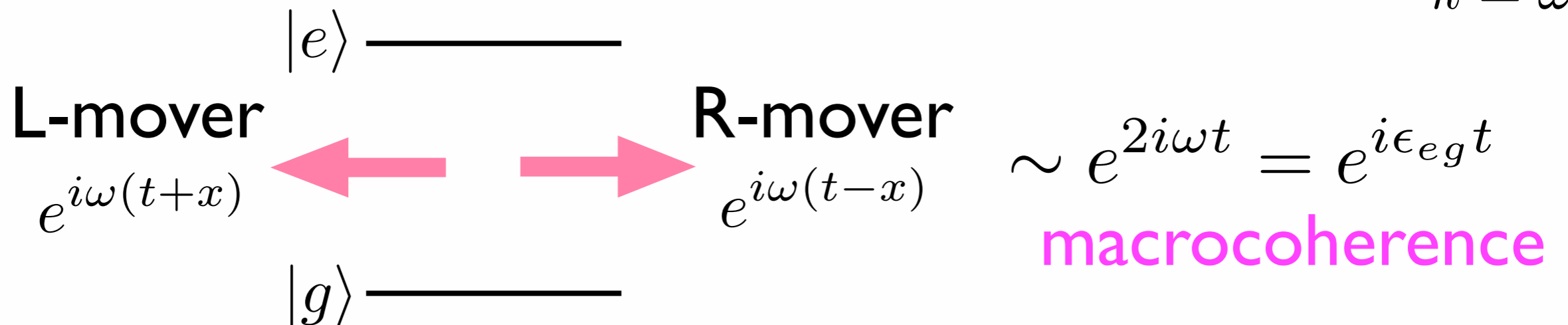
$$\alpha_{ge} = \frac{2d_{pe}d_{pg}}{\epsilon_{pg} + \epsilon_{pe}}, \quad \alpha_{aa} = \frac{2d_{pa}^2 \epsilon_{pa}}{\epsilon_{pa}^2 - \omega^2}, \quad (a = g, e)$$

d_{pa} : dipole matrix element

Field (1+1 dim.)

$$\omega = \epsilon_{eg}/2$$

$$E = E_R e^{-i(\omega t - kx)} + E_L e^{-i(\omega t + kx)} + \text{c.c.} \quad k = \omega$$



Bloch equation $\partial_t \rho = i[\rho, \mathcal{H}_I] + \text{relaxation terms}$
density matrix T_1, T_2

$$\rho = |\psi\rangle\langle\psi| = \rho_{gg}|g\rangle\langle g| + \rho_{ee}|e\rangle\langle e| + \rho_{eg}|e\rangle\langle g| + \rho_{ge}|g\rangle\langle e|$$

coherence (of an atom) $|\rho_{eg}| \leq 1/2$

Maxwell equation $(\partial_t^2 - \partial_x^2)E = -\partial_t^2 P$

macroscopic polarization $P = -\frac{\delta}{\delta E} \text{tr}(\rho \mathcal{H}_I)$

Rotating wave approximation (RWA)

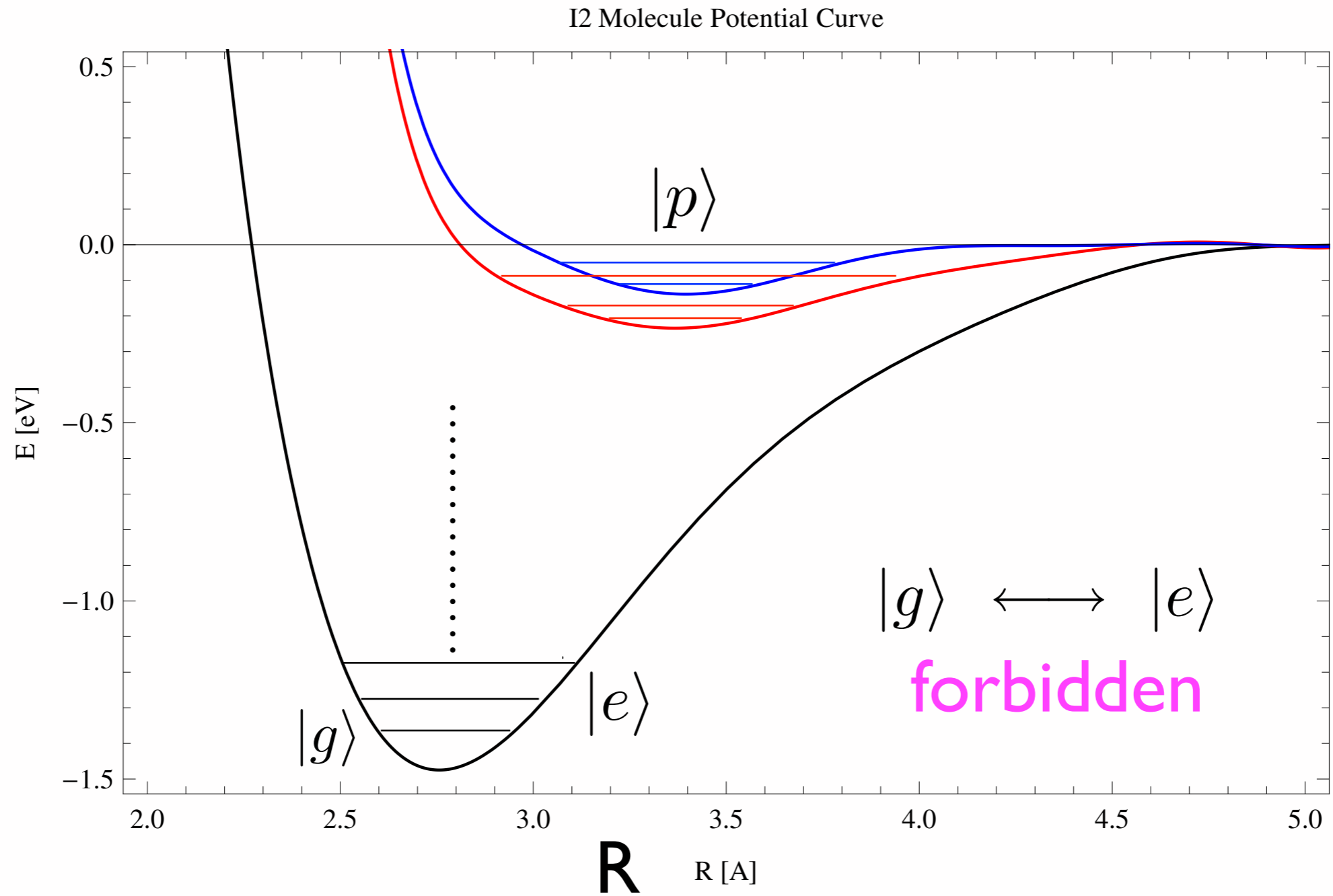
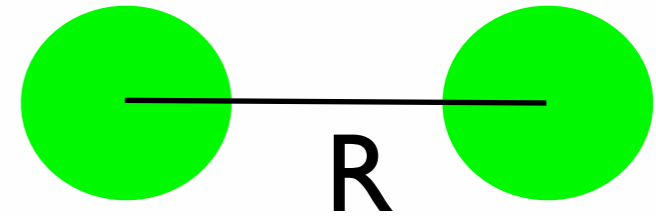
omitting fast oscillation terms

Slowly varying envelope approximation (SVEA)

$$|\partial_{x,t} E_{R,L}| \ll \omega |E_{R,L}|, \quad |\partial_{x,t} \rho| \ll \omega |\rho|$$

Homonuclear diatomic molecule

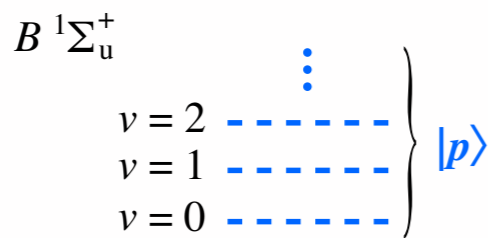
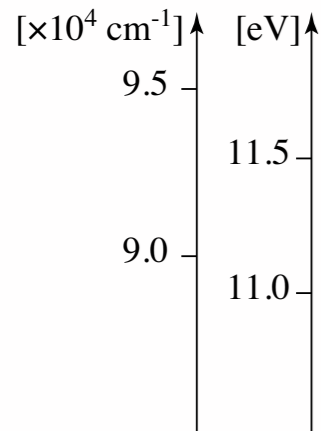
Potential curves



Numerical results

Target system: para-hydrogen molecule
gas or solid

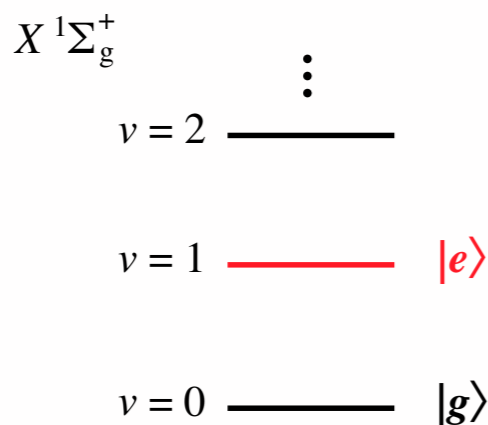
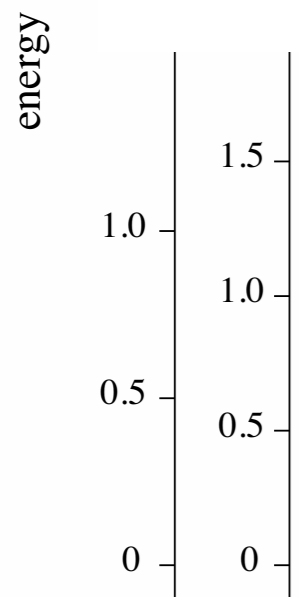
vibrational transition (electronic ground state)



$$|e\rangle = |X v = 1\rangle \longrightarrow |g\rangle = |X v = 0\rangle$$

no E1 transition

two-photon life $\sim 10^{11}$ sec.



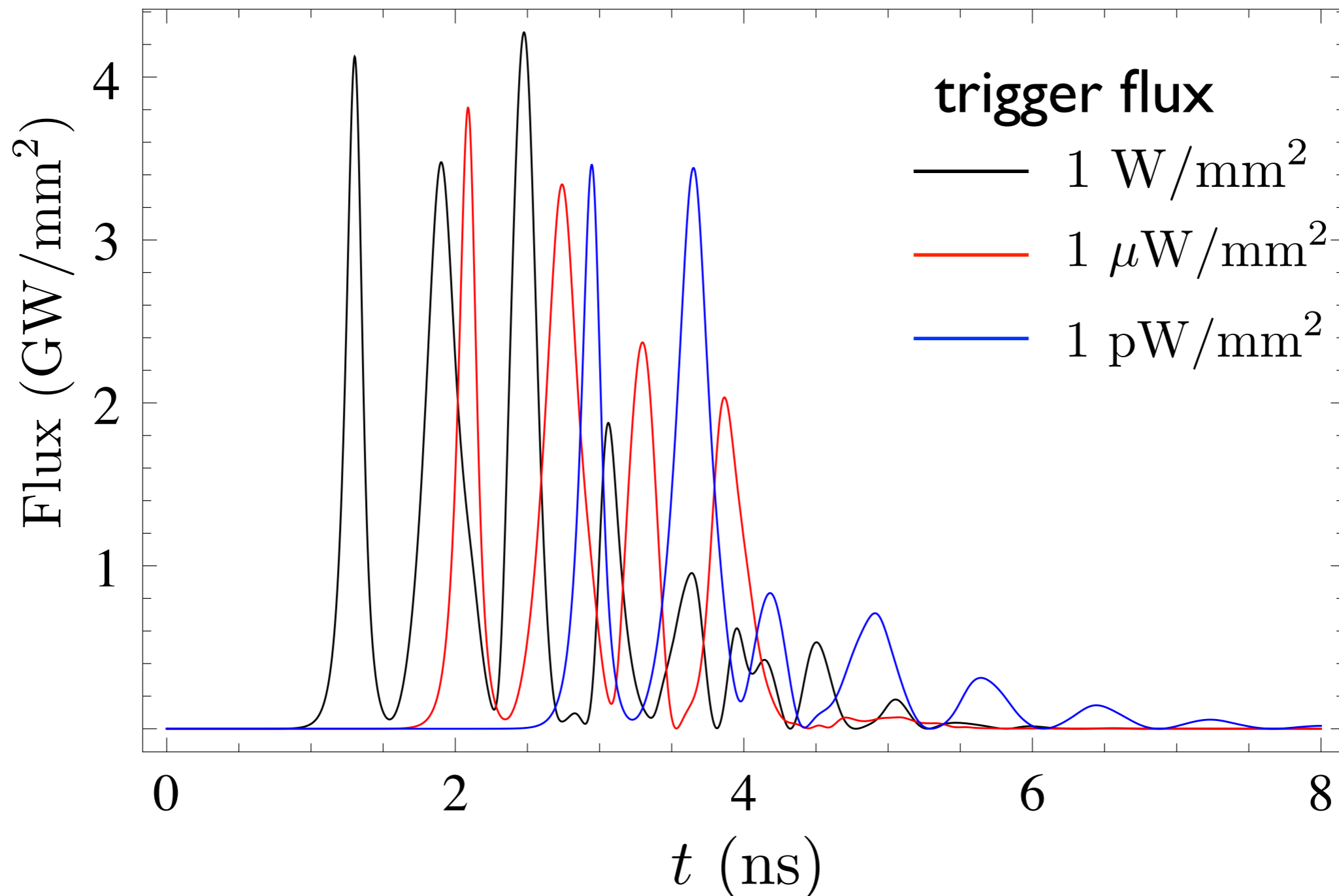
$$\epsilon_{eg} = 0.52 \text{ eV}, \quad \gamma_{\pm} = 15.3, 0.64$$

$$t_* \sim 50 \text{ ps} \frac{10^{21} \text{ cm}^{-3}}{n}$$

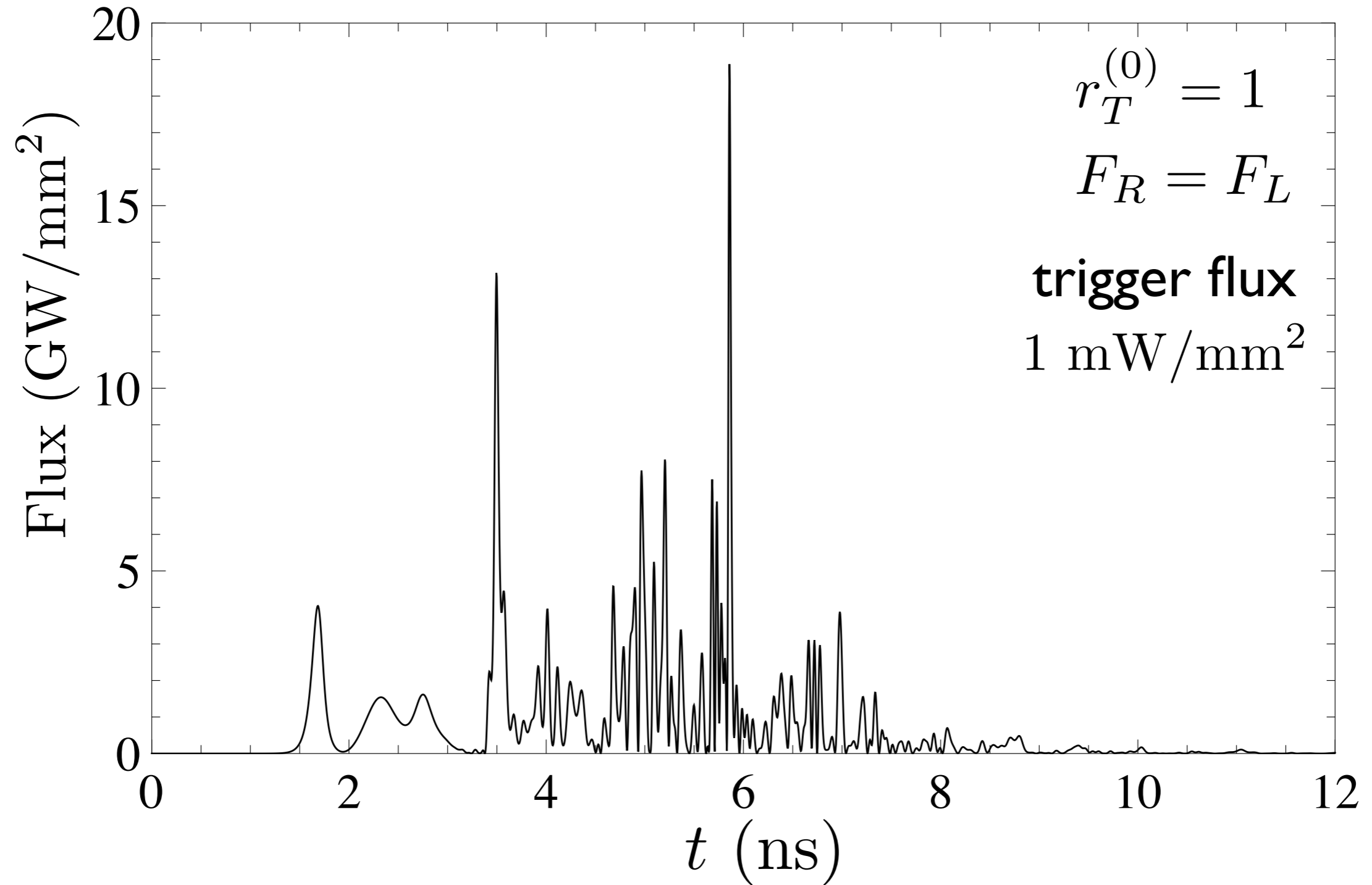
Explosive PSR

$$n = 1 \times 10^{21} / \text{cm}^3 \quad T_1 = 1 \mu\text{s}, \quad T_2 = 10 \text{ ns}$$

$$\text{target length } L = 30 \text{ cm} \quad r_T^{(0)} = 1 \quad (\rho_{eg} = 1/2)$$



$L = 100 \text{ cm}$



The dynamical factor

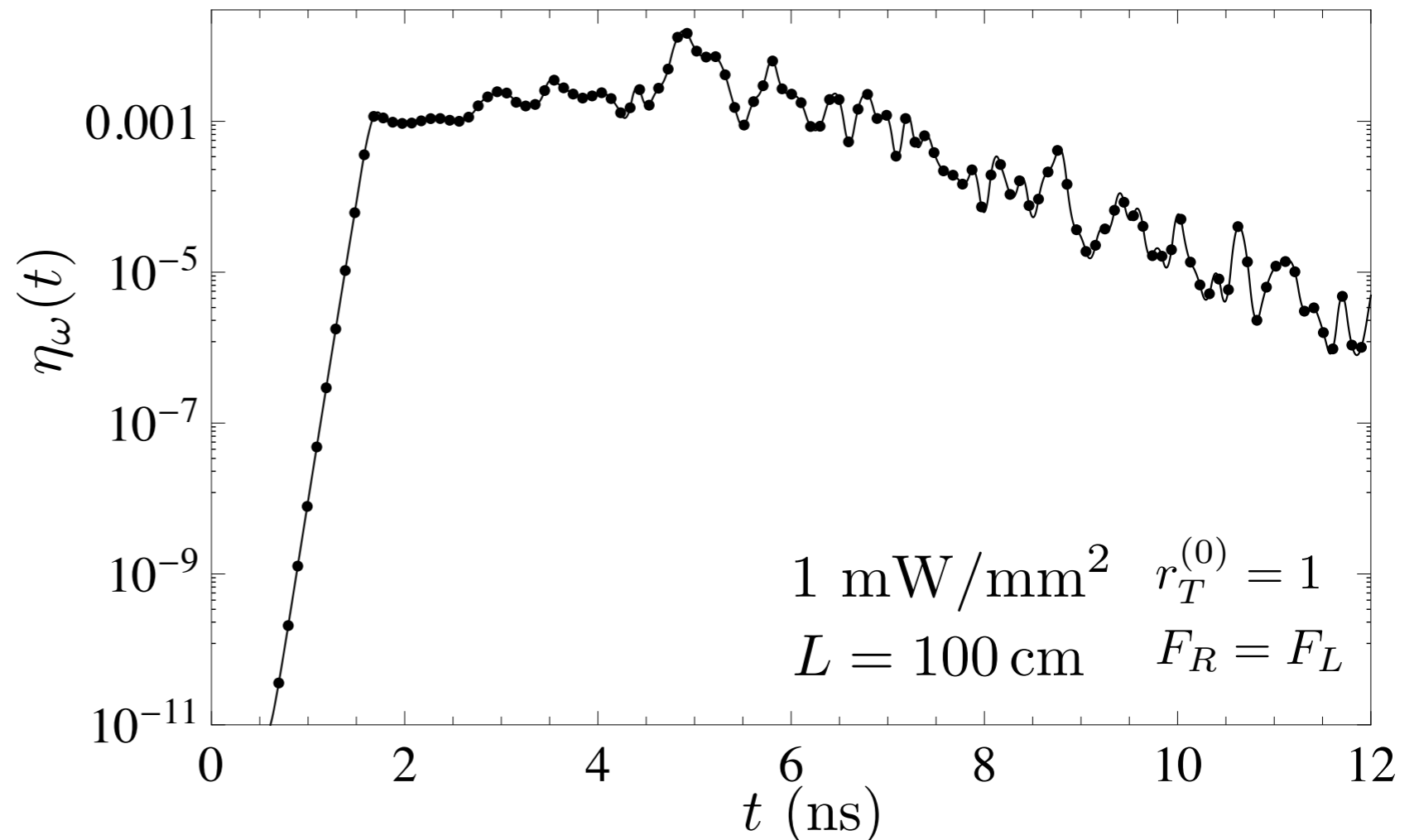
local field-medium activity

$$\eta_\omega(\xi, \tau) = \frac{1}{\epsilon_{eg} n^3} \left| \vec{E} - \frac{R_1 - iR_2}{2} \right|^2 = \left| \left(e_R^* e^{-ik\xi} + e_L^* e^{ik\xi} \right) \frac{r_1 - ir_2}{2} \right|^2$$

$$= \frac{1}{4} \left[(|e_R|^2 + |e_L|^2) (|r_T^{(0)}|^2 + |r_T^{(+)}|^2 + |r_T^{(-)}|^2) + 2\Re\{e_R^* e_L (r_T^{(0)*} r_T^{(+)} + r_T^{(0)} r_T^{(-)*})\} \right]$$

average over
the target length

$$\eta_\omega(t) = \langle \eta_\omega(\xi, \tau) \rangle_\xi$$



PSR with spatial gratings

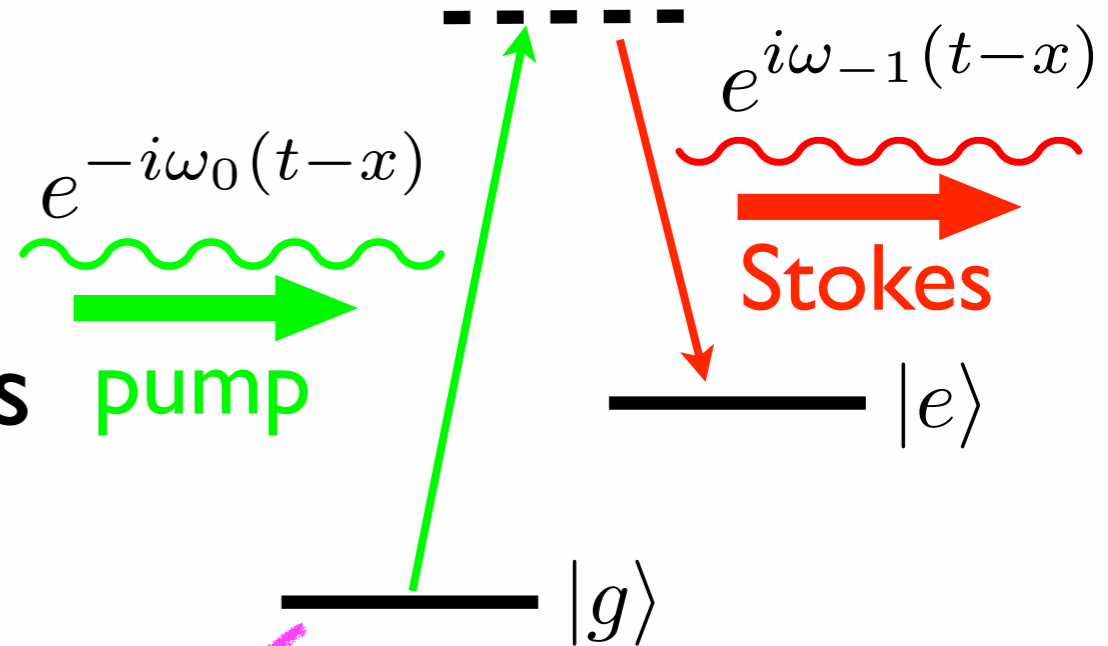
How to populate $|e\rangle$

Stimulated Raman process

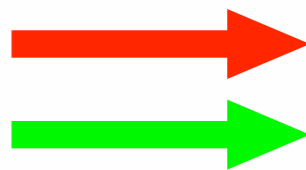
$$\omega_0 - \omega_{-1} = \epsilon_{eg}$$

Generated coherence

$$\rho_{eg} = \rho_{eg}^{(0)} + \rho_{eg}^{(+)} e^{i\epsilon_{eg}x} + \rho_{eg}^{(-)} e^{-i\epsilon_{eg}x}$$



Stokes
pump



ω_p PSR
 $\omega_{\bar{p}}$

$$e^{i\omega_p(t-x)} e^{i\omega_{\bar{p}}(t-x)} = e^{i\epsilon_{eg}(t-x)}$$

$$\omega_p + \omega_{\bar{p}} = \epsilon_{eg}$$

momentum conservation
in the macrocoherence

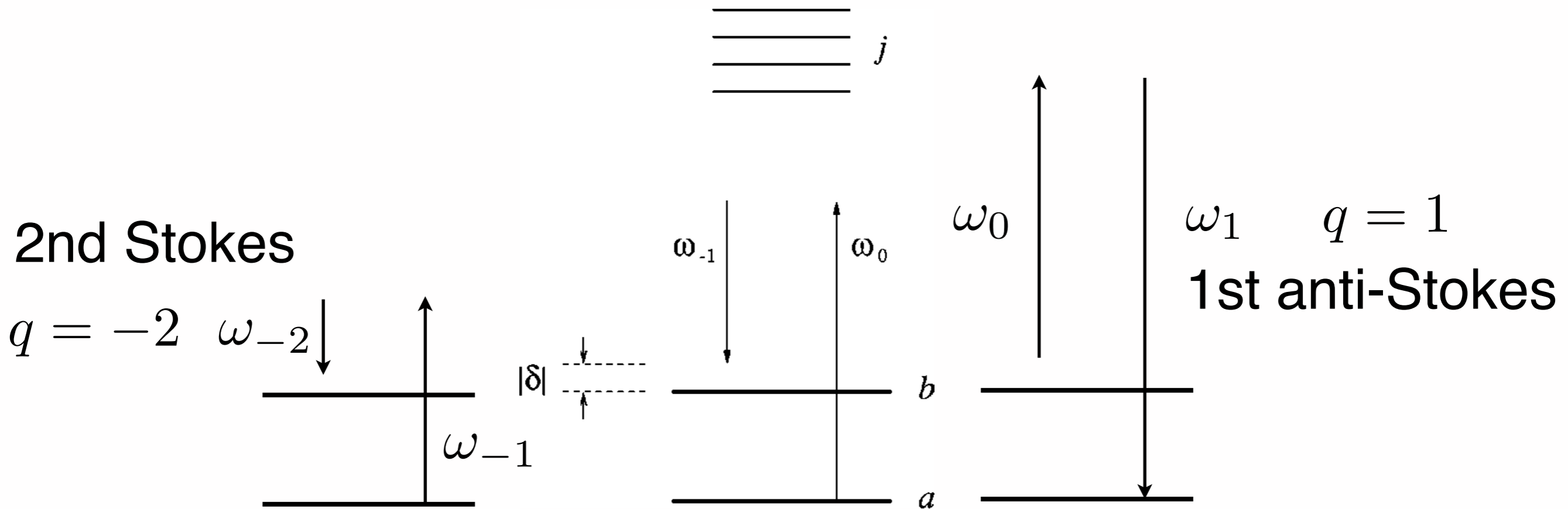


Unidirectional PSR

Raman sideband generation

Harris, Sokolov, Phys. Rev.A55, R4019(1997)

Kien, Liang, Katsuragawa, Ohtsuki, Hakuta, Sokolov, Phys. Rev.A60, 1562(1999)



$$\omega_q = \omega_0 + q(\omega_0 - \omega_{-1}) = \omega_0 + q(\omega_b - \omega_a - \delta)$$

$q \geq q_{\min}$ the lowest Stokes

↑
detuning

Hamiltonian

$$H_{\text{int}} = - \sum_j E (\mu_{ja} \sigma_{ja} + \mu_{aj} \sigma_{aj} + \mu_{jb} \sigma_{jb} + \mu_{bj} \sigma_{bj})$$

$$\mu_{\alpha\beta} = \langle \alpha | d | \beta \rangle \quad \sigma_{\alpha\beta} = |\alpha\rangle\langle\beta|$$

$$E = \frac{1}{2} \sum_q (E_q e^{-i\omega_q \tau} + E_q^* e^{i\omega_q \tau})$$

Effective Hamiltonian

$|j\rangle$ far off-resonance  two-level system

$$H_{\text{eff}} = -\hbar \begin{bmatrix} \Omega_{aa} & \Omega_{ab} \\ \Omega_{ba} & \Omega_{bb} - \delta \end{bmatrix}$$

Stark shifts

$$\Omega_{aa} = \frac{1}{2} \sum_q a_q |E_q|^2 \quad a_q = \frac{1}{2\hbar^2} \sum_j \left(\frac{|\mu_{ja}|^2}{\omega_j - \omega_a - \omega_q} + \frac{|\mu_{ja}|^2}{\omega_j - \omega_a + \omega_q} \right)$$

$$\Omega_{bb} = \frac{1}{2} \sum_q b_q |E_q|^2 \quad b_q = \frac{1}{2\hbar^2} \sum_j \left(\frac{|\mu_{jb}|^2}{\omega_j - \omega_b - \omega_q} + \frac{|\mu_{jb}|^2}{\omega_j - \omega_b + \omega_q} \right)$$

Two-photon Rabi freq.

$$\Omega_{ab} = \Omega_{ba}^* = \frac{1}{2} \sum_q d_q E_q E_{q+1}^* \quad d_q = \frac{1}{2\hbar^2} \sum_j \left(\frac{\mu_{aj}\mu_{jb}}{\omega_j - \omega_b - \omega_q} + \frac{\mu_{aj}\mu_{jb}}{\omega_j - \omega_a + \omega_q} \right)$$

Adiabatic eigenstate

$$|+\rangle = \cos \frac{\theta}{2} e^{i\varphi/2} |a\rangle + \sin \frac{\theta}{2} e^{-i\varphi/2} |b\rangle \xrightarrow{E \rightarrow 0} |a\rangle$$

$$\tan \theta = \frac{2|\Omega_{ab}|}{\Omega_{aa} - \Omega_{bb} + \delta} \quad \Omega_{ab} = |\Omega_{ab}| e^{i\varphi}$$

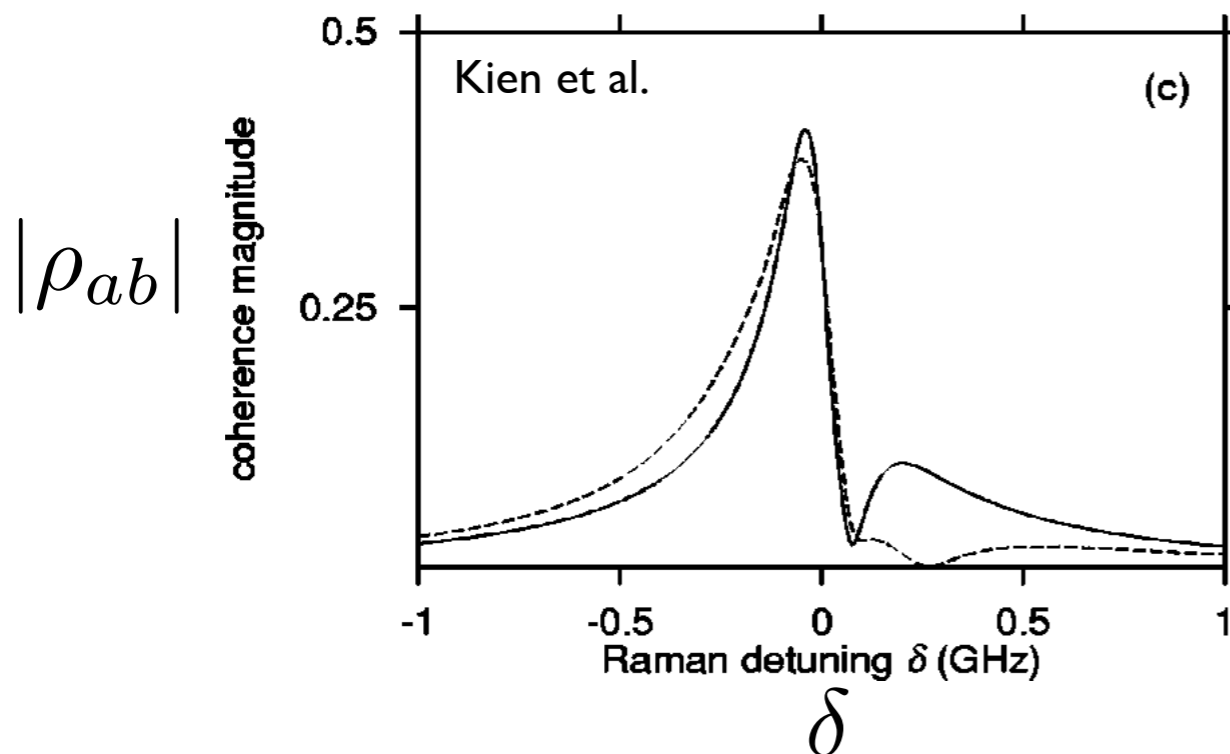
Wave propagation

$$(\partial_t + \partial_z)E_q = in\hbar\omega_q (a_q\rho_{aa}E_q + b_q\rho_{bb}E_q + d_{q-1}\rho_{ba}E_{q-1} + d_q^*\rho_{ab}E_{q+1})$$

Coherence $\rho_{ab} = \frac{1}{2} \sin \theta e^{i\varphi}$

molecular system of far off-resonance

$$\Omega_{aa} \simeq \Omega_{bb} \quad \tan \theta \simeq 2|\Omega_{ab}|/\delta \quad \longrightarrow \quad |\rho_{ab}| \simeq 1/2$$



$$\delta > 0, \sin \theta > 0$$

phased state

$$\delta < 0, \sin \theta < 0$$

antiphased state

PSR experiments

Para-hydrogen gas PSR experiment

@ Okayama U

Y. Miyamoto et al. PTEPI 13C01 (2014)

arXiv 1406.2198

vibrational transition of p-H₂

$$|e\rangle = |Xv = 1\rangle \longrightarrow |g\rangle = |Xv = 0\rangle$$

two-photon decay: $\tau_{2\gamma} \sim 10^{11}$ s

p-H₂: nuclear spin=singlet
smaller decoherence 遅い

$$1/T_2 \sim 130 \text{ MHz}$$

coherence production

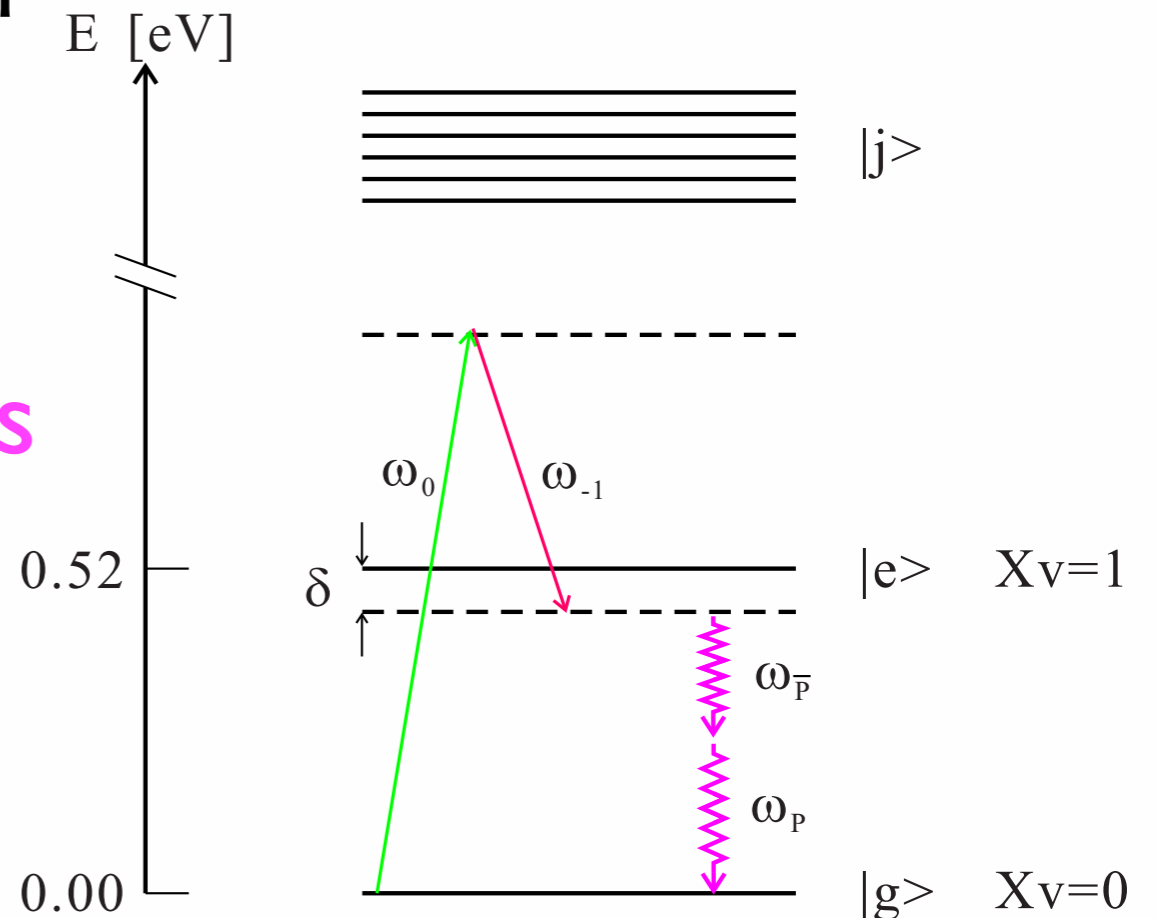
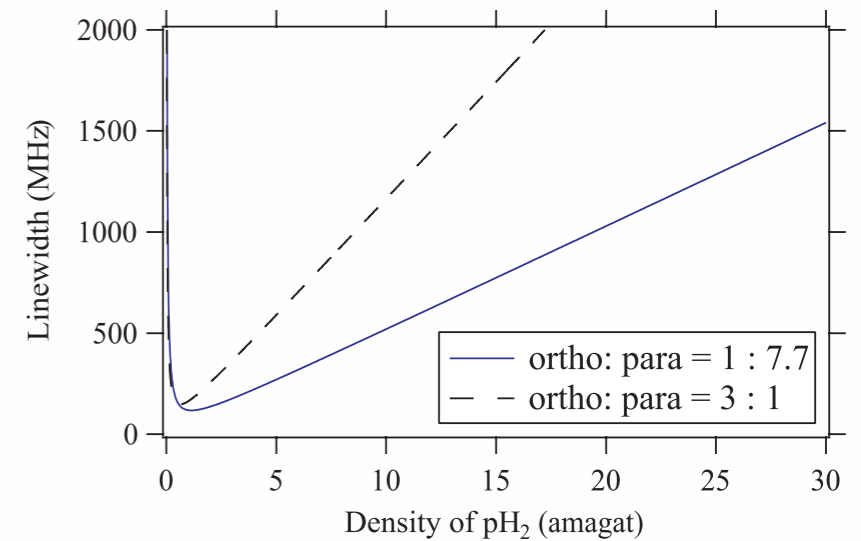
adiabatic Raman process

$$\Delta\omega = \omega_0 - \omega_{-1}$$

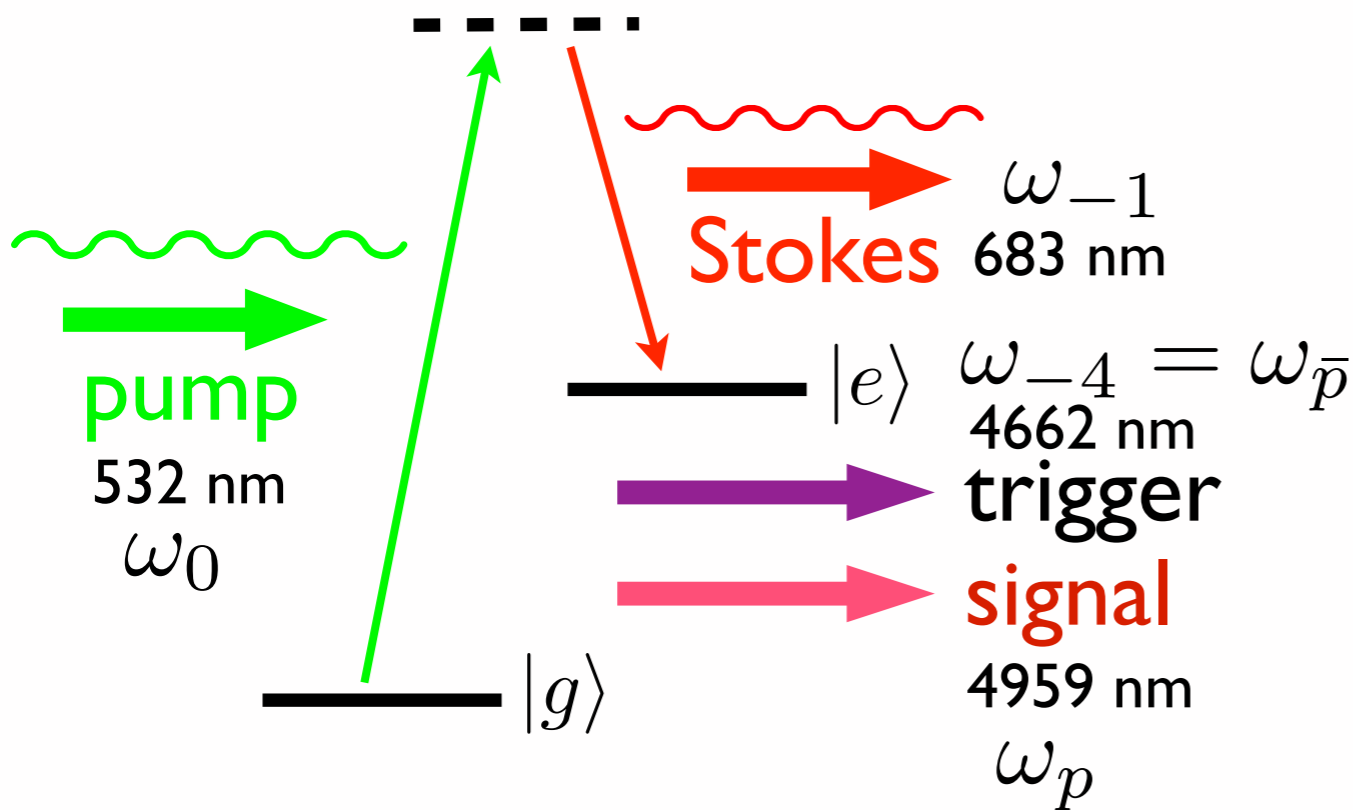
$$= \epsilon_{eg} - \delta$$

$$= \omega_p + \omega_{\bar{p}}$$

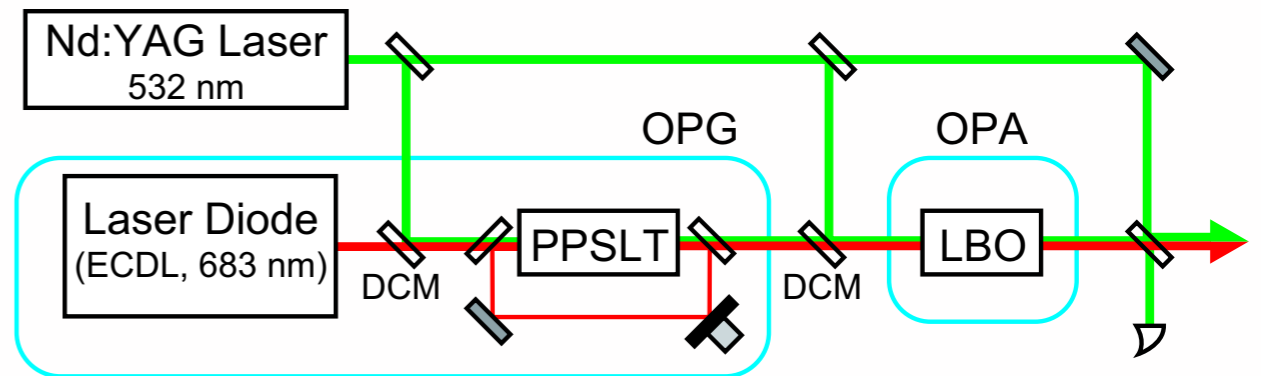
detuning



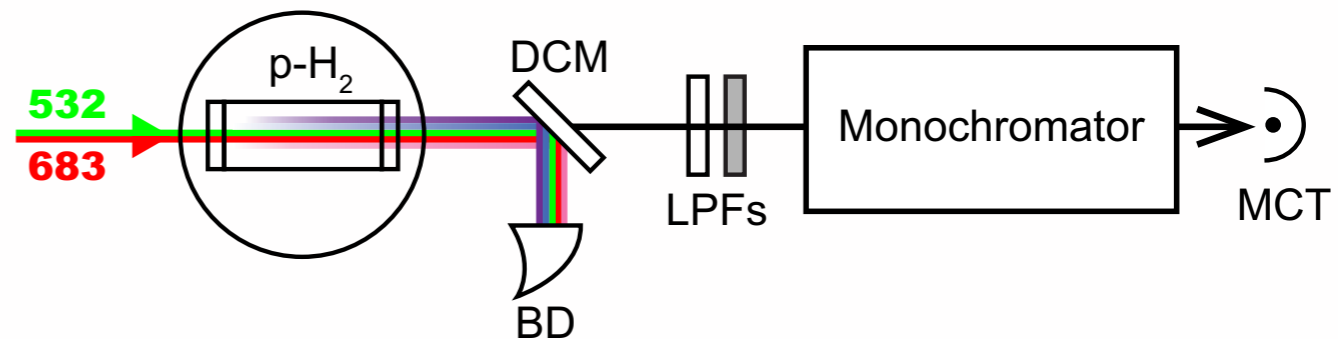
Experimental setup



(a) Laser Setup



(b) Target & Detector



Driving lasers: 5 mJ , 6 ns , $w_0 = 100\ \mu\text{m}$ ($5\text{ GW}/\text{cm}^2$)

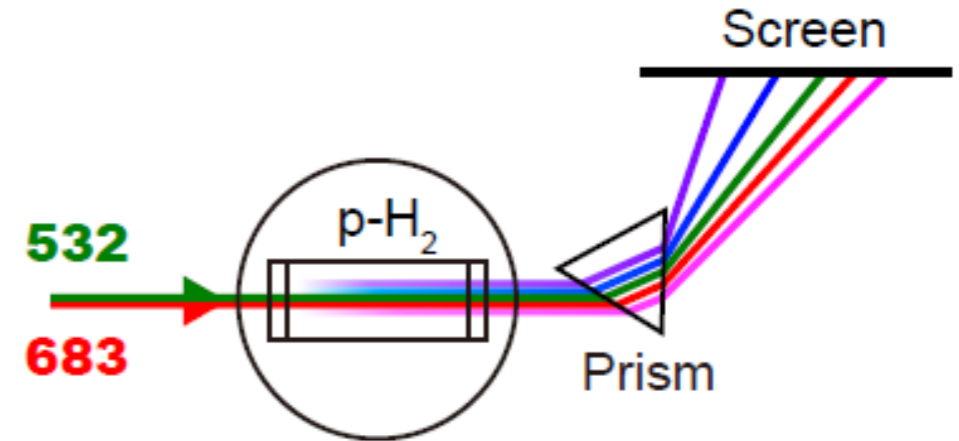
Target cell: length 15 cm , diameter 2 cm , 78 K , 60 kPa

$$n = 5.6 \times 10^{19}\text{ cm}^{-3} \quad 1/T_2 \sim 130\text{ MHz}$$

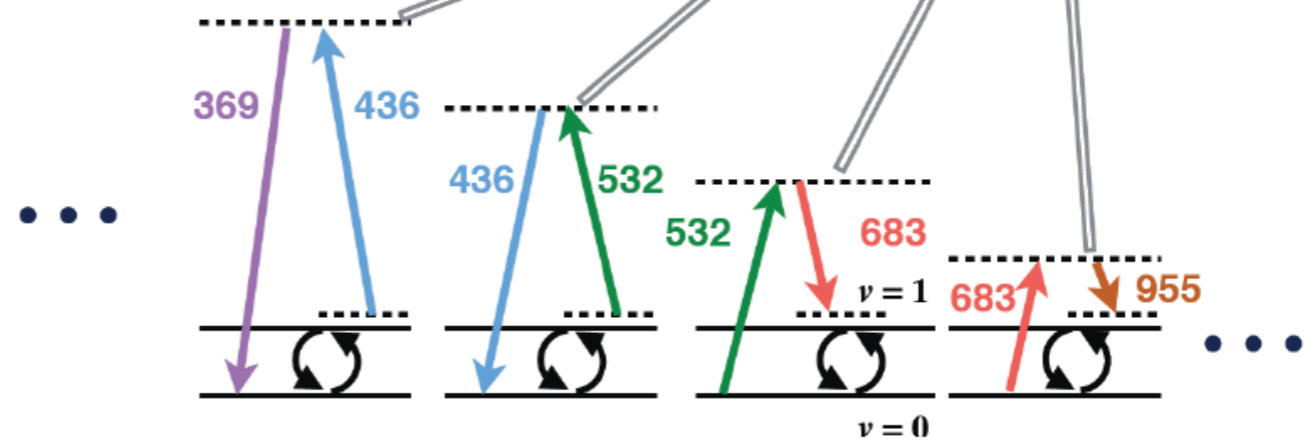
4th Stokes ($q=-4$) as trigger (internal trigger)

Ultra-broadband Raman sidebands

- Raman sidebands, from 192 to 4662nm, are observed: >24
- Evidence of large coherence



2014/10/29



Kyoto

34

N. Sasao

Generated coherence

Maxwell-Bloch eq.

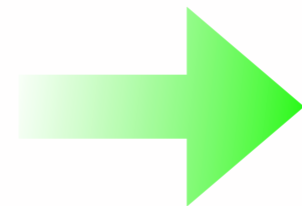
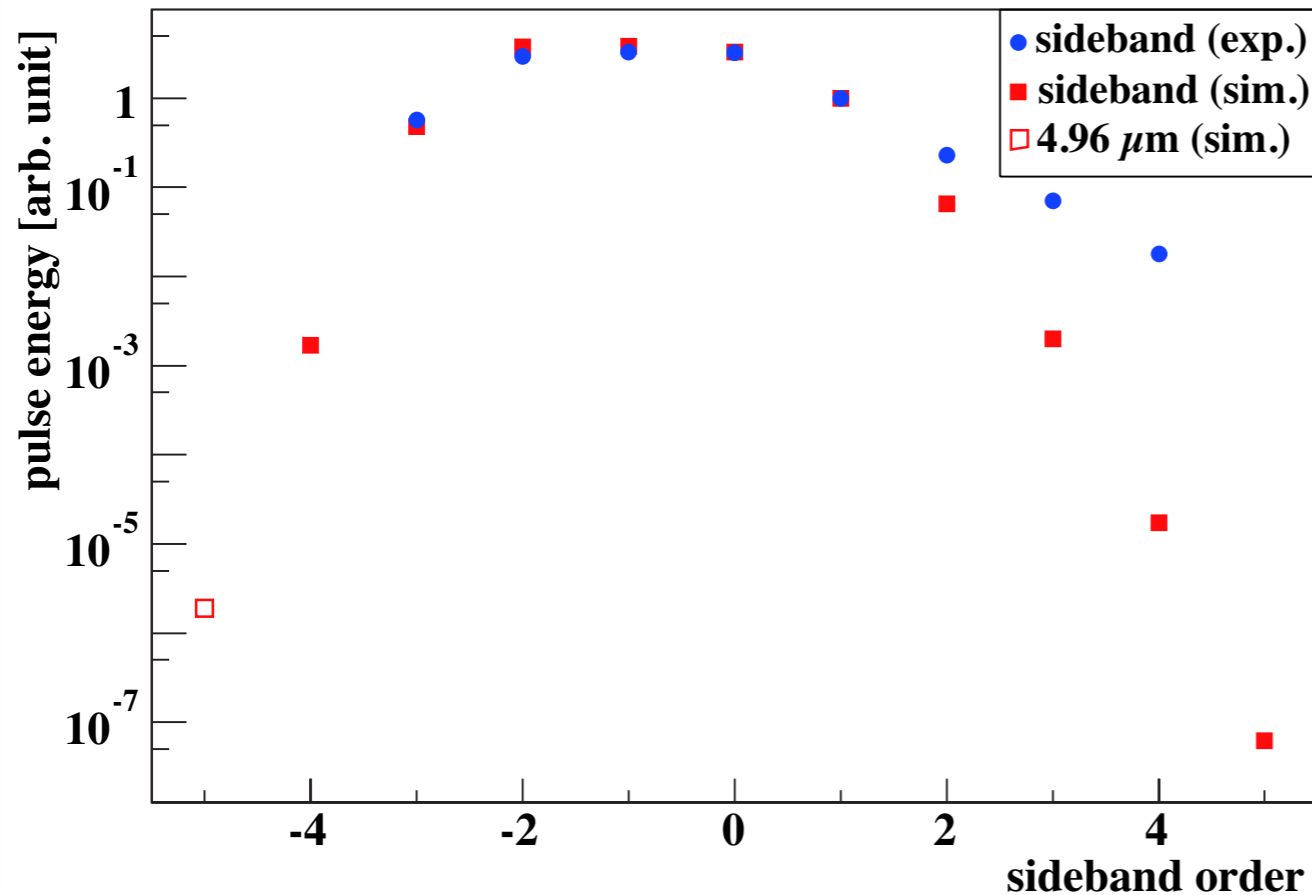
$$\frac{\partial \rho_{gg}}{\partial \tau} = i(\Omega_{ge}\rho_{eg} - \Omega_{eg}\rho_{ge}) + \gamma_1\rho_{ee},$$

$$\frac{\partial \rho_{ee}}{\partial \tau} = i(\Omega_{eg}\rho_{ge} - \Omega_{ge}\rho_{eg}) - \gamma_1\rho_{ee},$$

$$\frac{\partial \rho_{ge}}{\partial \tau} = i(\Omega_{gg} - \Omega_{ee} + \delta)\rho_{ge} + i\Omega_{ge}(\rho_{ee} - \rho_{gg}) - \gamma_2\rho_{ge},$$

$$\frac{\partial E_q}{\partial \xi} = \frac{i\omega_q n}{2c} \left\{ (\rho_{gg}\alpha_{gg}^{(q)} + \rho_{ee}\alpha_{ee}^{(q)})E_q + \rho_{eg}\alpha_{eg}^{(q-1)}E_{q-1} + \rho_{ge}\alpha_{ge}^{(q)}E_{q+1} \right\},$$

$$\frac{\partial E_p}{\partial \xi} = \frac{i\omega_p n}{2c} \left\{ (\rho_{gg}\alpha_{gg}^{(p)} + \rho_{ee}\alpha_{ee}^{(p)})E_p + \rho_{eg}\alpha_{ge}^{(p\bar{p})}E_{\bar{p}}^* \right\}.$$

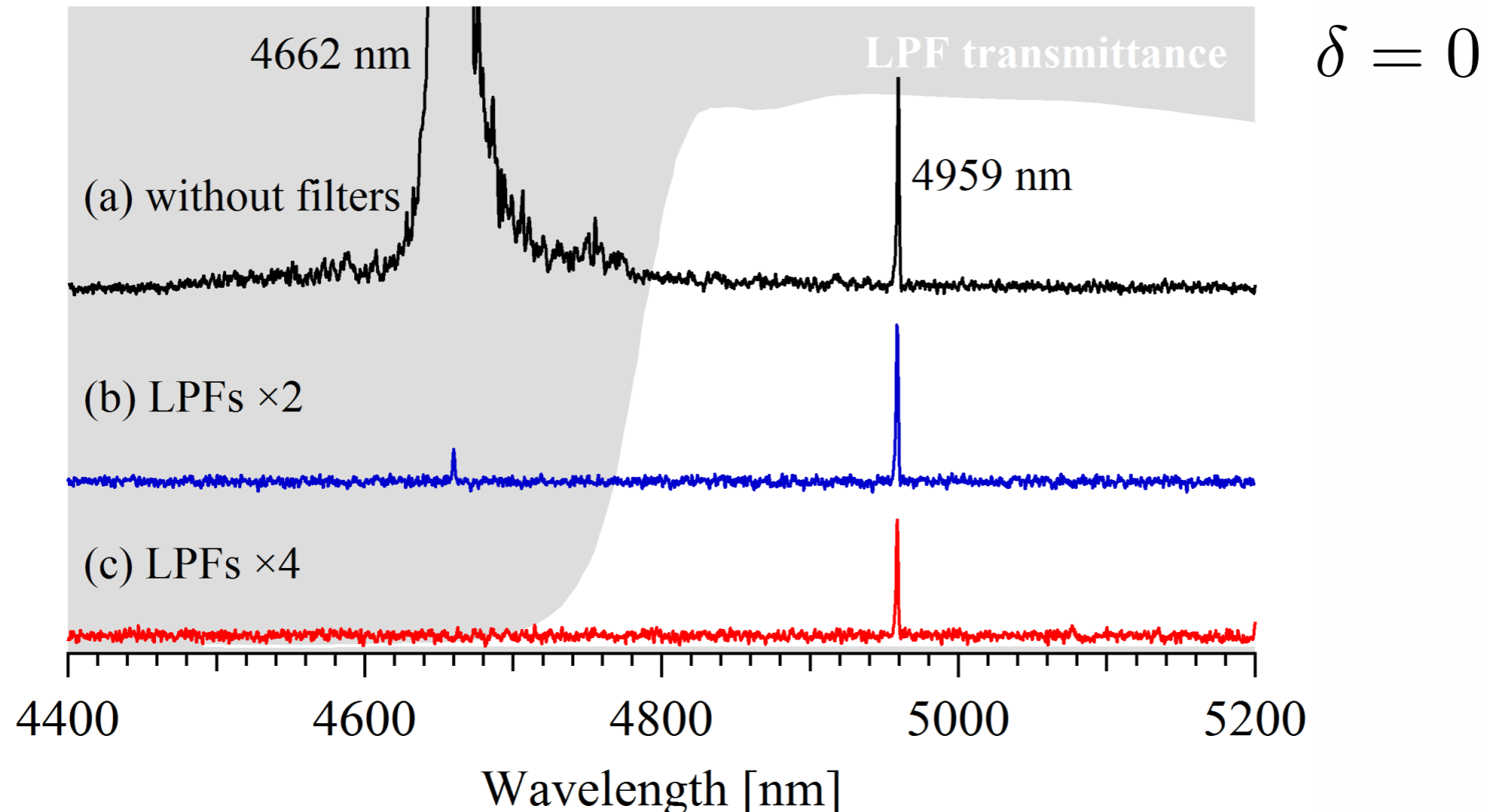


coherence estimation

$$|\rho_{eg}| \simeq 0.032$$

(6% of max.)

Observed two-photon spectrum

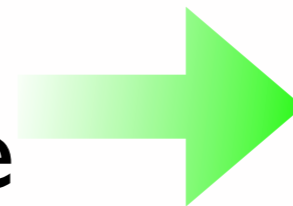


of observed photons

$$4.4 \times 10^7 / \text{pulse}$$

Estimated spontaneous rate

$$1.6 \times 10^{-8}$$



$O(10^{15})$ (or more)
enhancement!

Experiment with an external trigger

Y. Miyamoto et al. arXiv1505.07663, to appear in PTEP

Internal trigger (4th Stokes)

$$\omega_q = \omega_0 + q(\omega_0 - \omega_{-1}), \quad q = -4$$

frequency, power, timing tied to driving lasers

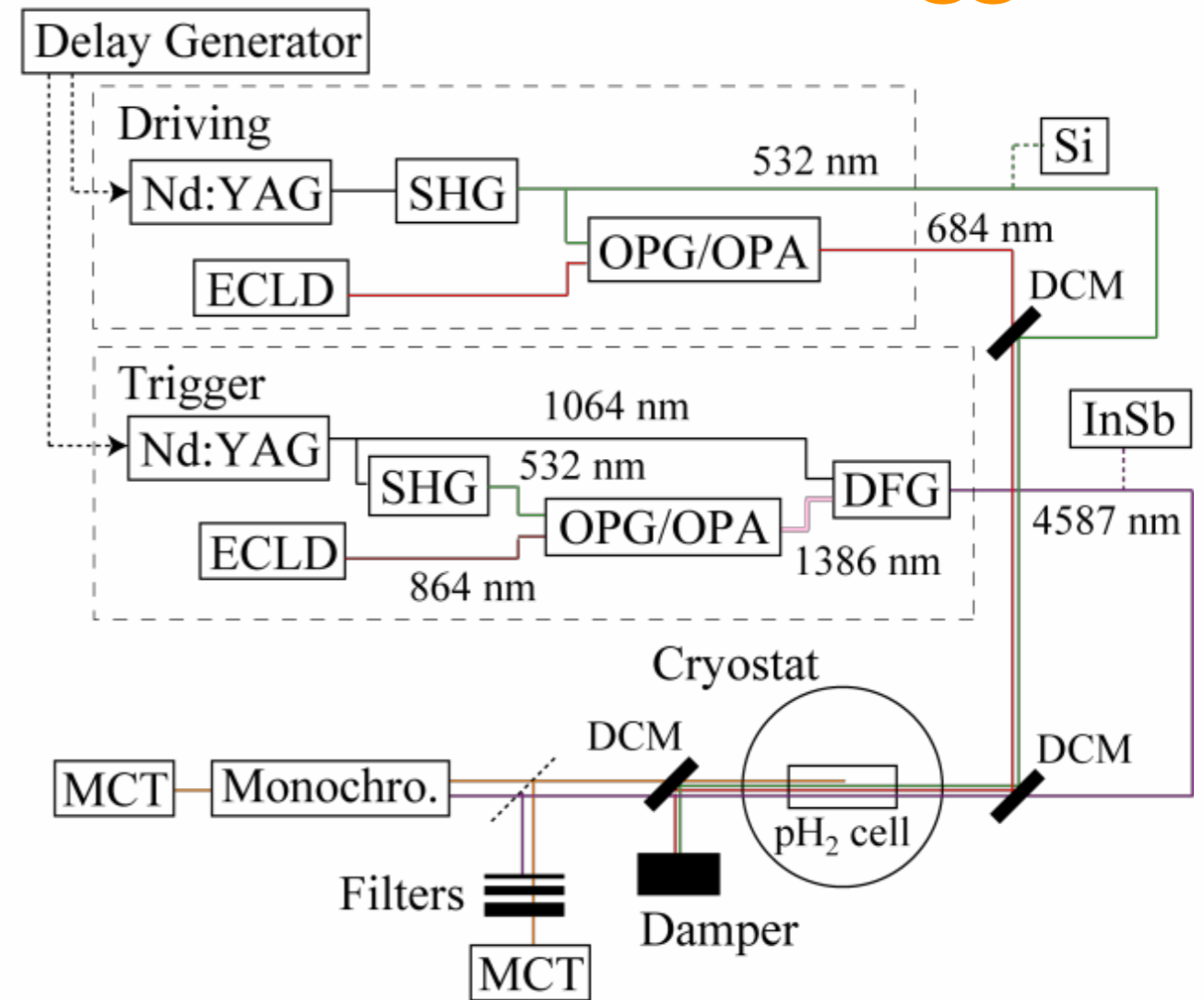
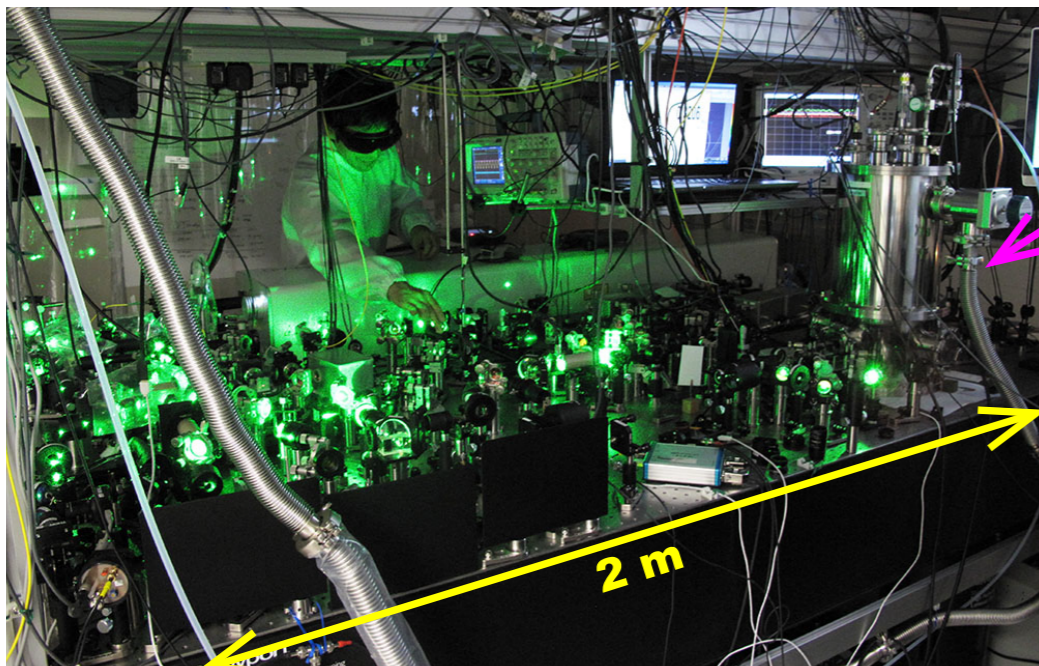
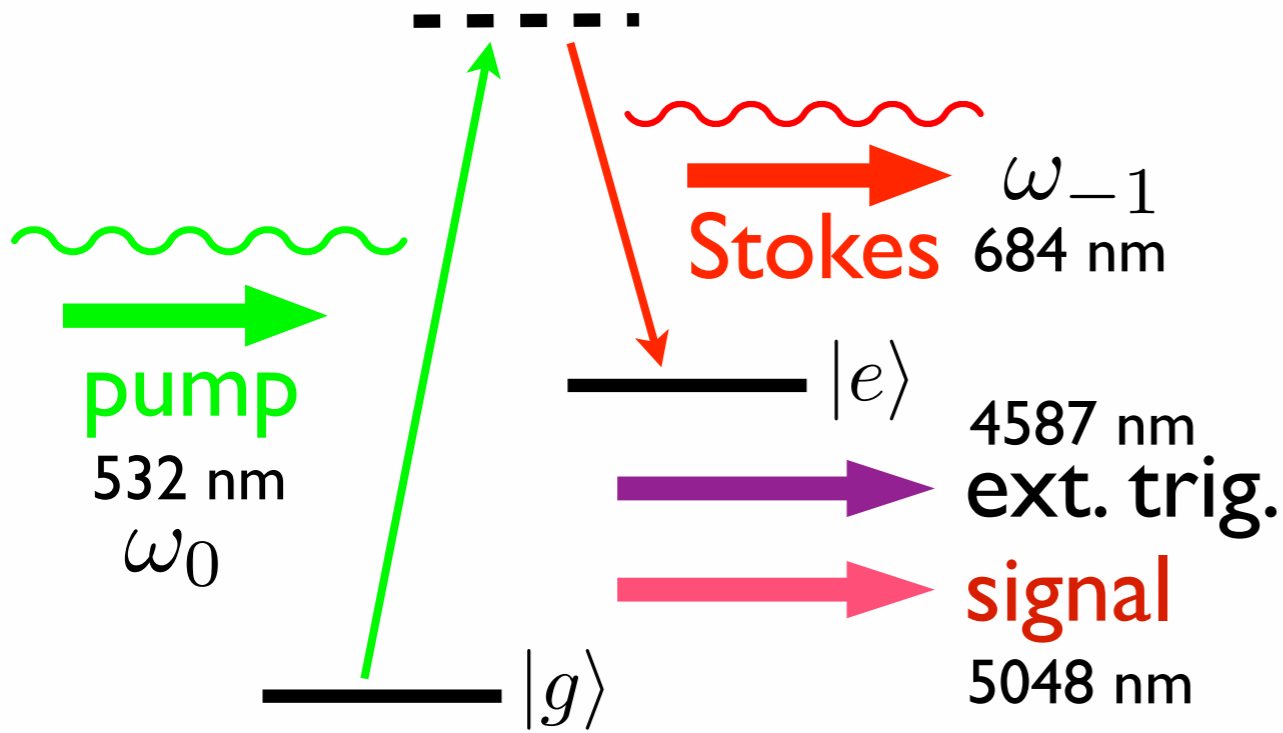
External trigger (additional laser irradiation)

frequency, power, timing indep. of driving lasers

 More suitable to explore macrocoherence

Mandatory for RENP

Experimental setup with an external trigger

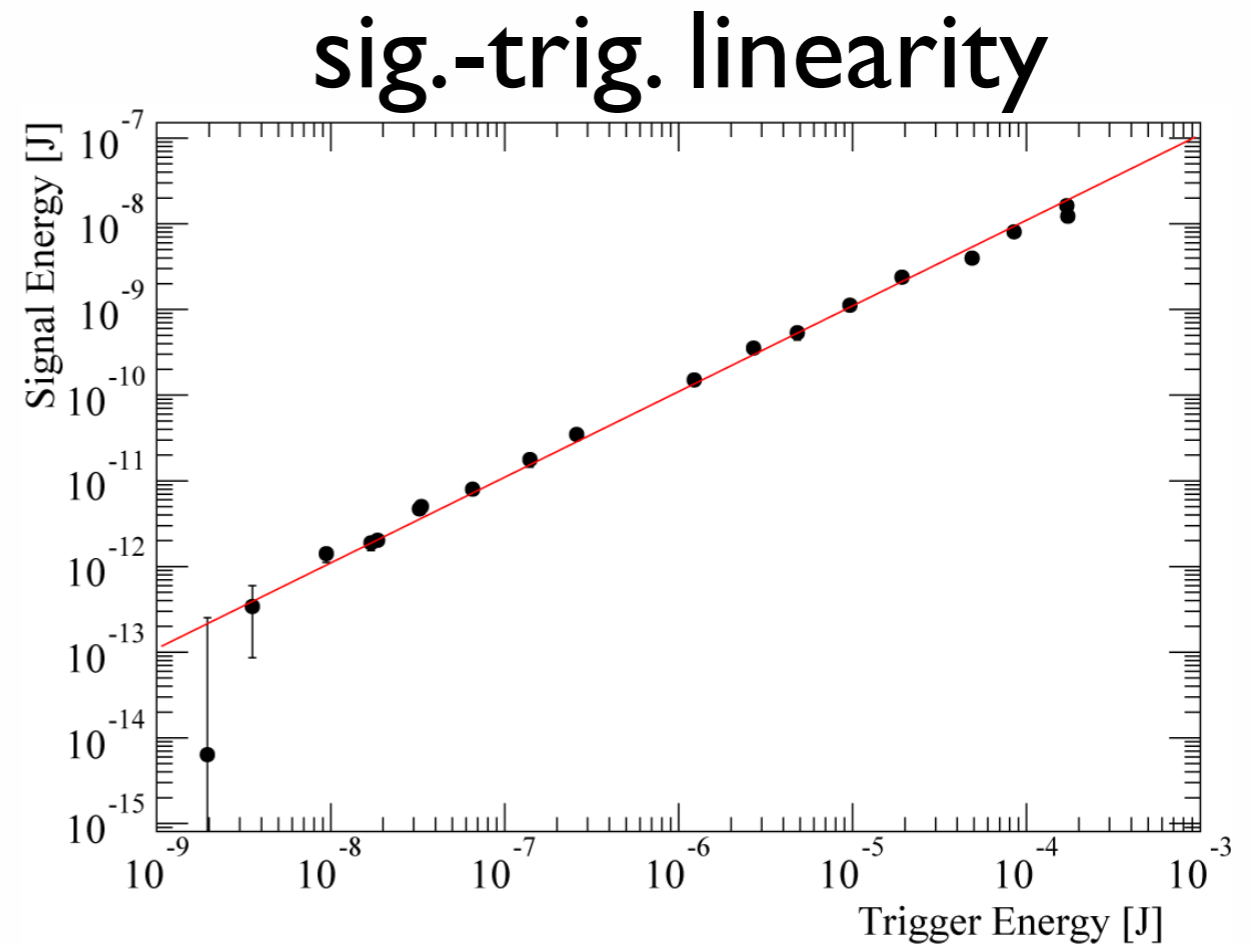
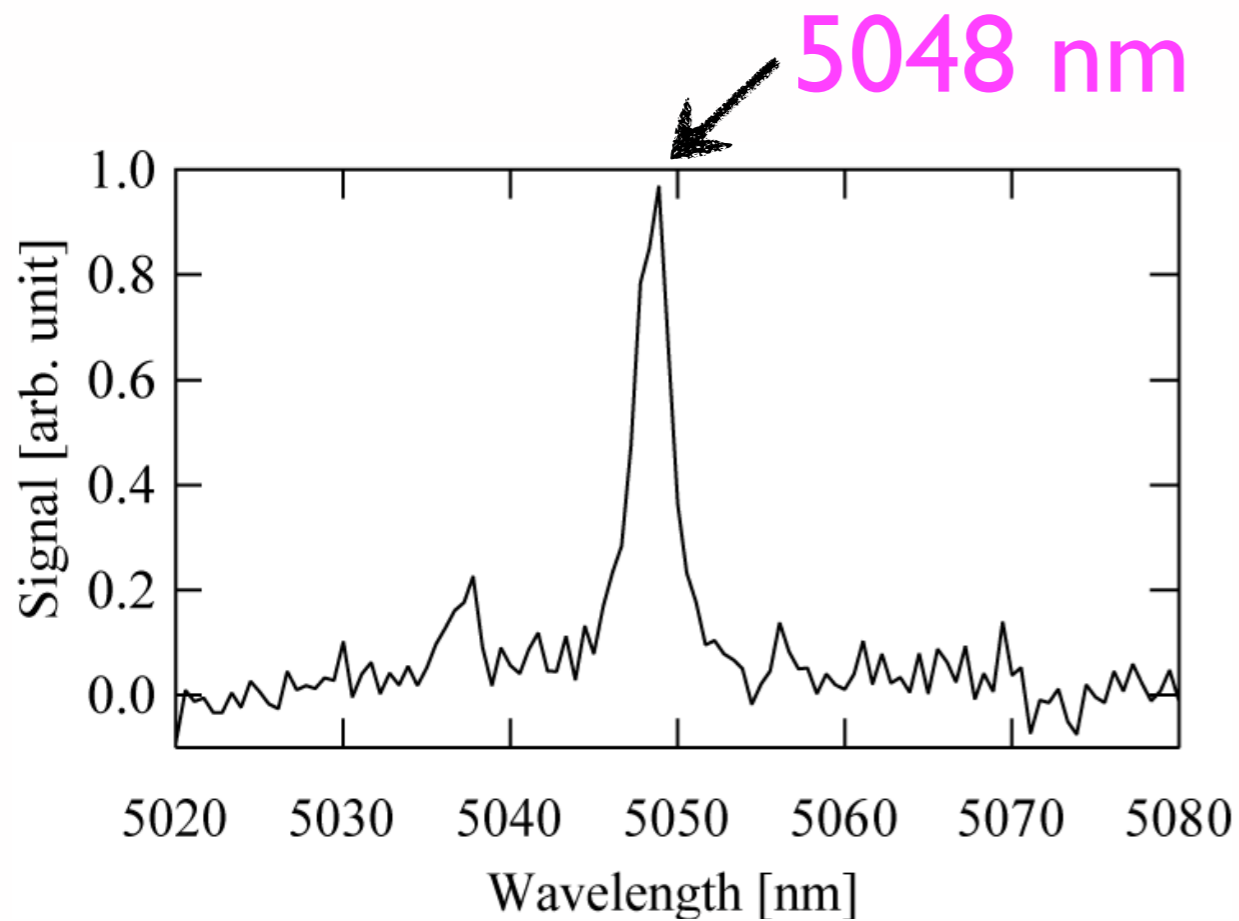


External trigger
4587 nm, 2 ns, 150 $\mu\text{J}/\text{pulse}$

Results

Estimated coherence (from sidebands)

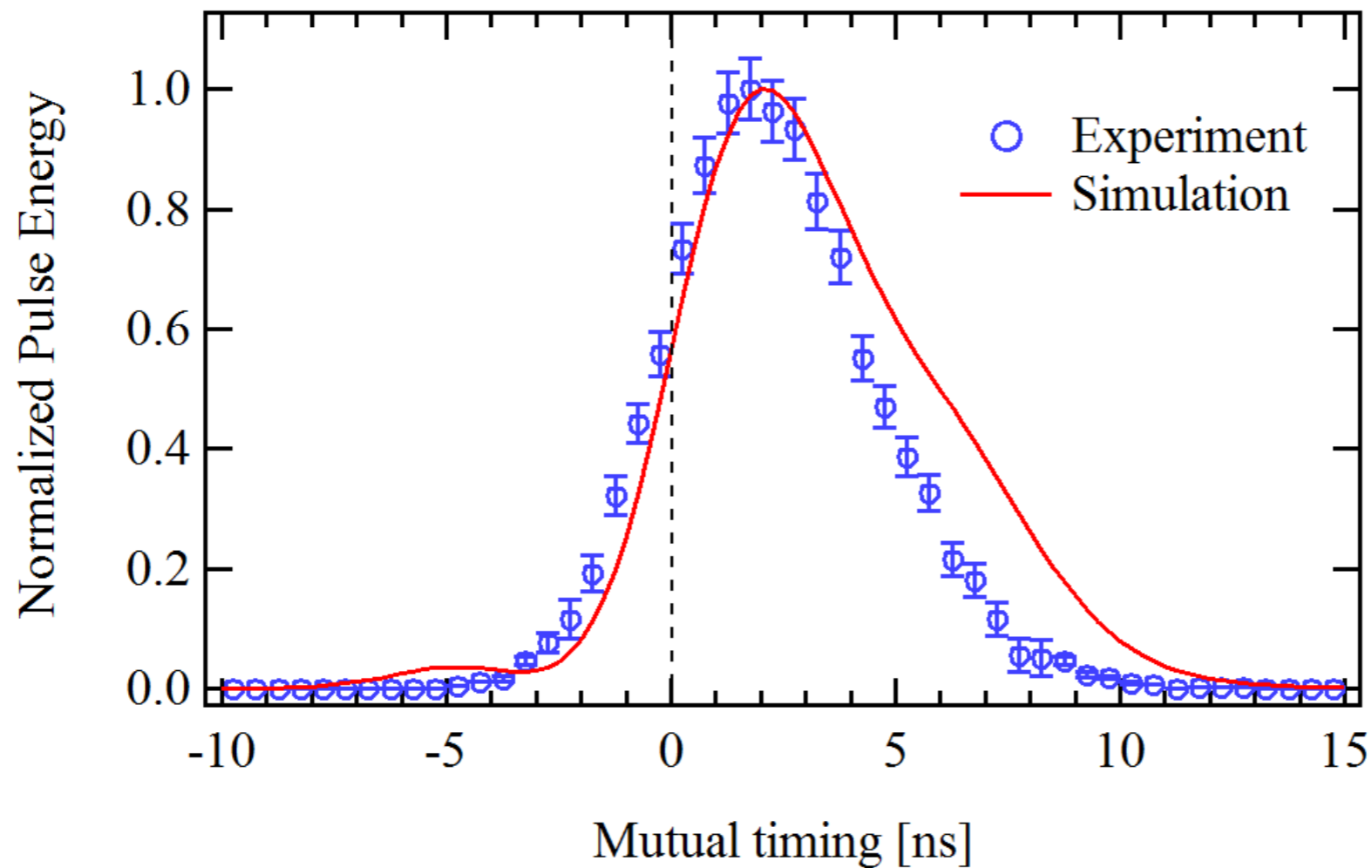
$$|\rho_{eg}| \sim 0.04 \quad (\delta = -160 \text{ MHz})$$



6×10^{11} photons/pulse
→ 10^{18} enhancement

weak field
low coherence

Trigger timing



trigger delay

Delayed coherence development in the target

SUMMARY

Coherences

Atomic coherence ρ_{eg}

Target coherence $\left[(|g\rangle + |e\rangle) / \sqrt{2} \right]^N$

Macrocoherence $e^{i\omega(t-x)} e^{i\omega(t+x)} = e^{2i\omega t}$

Realized in adiabatic Raman process
with (internal or external) trigger



p-H₂ PSR experiments

Huge **enhancement** confirmed.